Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Heat and Mass Transfer 48 (2005) 5072–5077

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

Heat transfer of a viscoelastic fluid in a porous channel

C. Kurtcebe *, M.Z. Erim

Department of Aeronautical Engineering, Faculty of Aeronautics and Astronautics, Istanbul Technical University, 80626 Maslak, 34469 Istanbul, Turkey

Received 23 January 2004; received in revised form 18 May 2005
Available online 24 August 2005

1. Introduction

Non-Newtonian fluid flows are encountered in a wide range of engineering applications. Hot rolling, extrusion of plastics, flow in journal bearings, lubrication, flow in a shock absorber are some typical examples to name just a few [1,2]. The increase of these applications in the past decades have urged scientists and engineers to provide mathematical models for non-Newtonian fluids. The nonlinearity between stress and deformation rate for this kind of fluids makes it, in general, impossible to obtain a simple mathematical model as in the case for Newtonian fluids. This difficulty has lead researchers to investigate relatively simple non-Newtonian fluid models. In this sense, simple viscometric flows have been extensively studied to date, which include a number of problems of engineering interest [1–3]. For viscoelastic fluids, which are also known as second-order fluids, a simple model is proposed by Rivlin and Ericksen [4,5]. This model predicts normal stress effects but it maintains Newtonian viscosity. Therefore, its range of applicability is limited to very low deformation rates or to materials that are only slightly viscoelastic. For flows with moderate or high deformations rates it is required to use a third-order model or a variable viscosity model proposed by White and Metzner [6], who assumed the Newtonian viscosity term to depend on these invariants of the acceleration tensors. A detailed discussion on second and third-order fluids can be found in the study of Dunn and Rajagopal [7].

Based on these models several studies concerning simple flow problems have been published; namely, the flow near a stagnation point has been investigated by Srivastava [8] in the case of viscoinelastic fluid. Rajeswari and Rathna [9] extended the work of Srivastava to two and three dimensional viscoelastic and viscoinelastic fluid flow. Among more recent publications, Raghay and Hakim [10] have developed a finite volume technique for the simulation of viscoelastic flow using White–Metzner fluid model. Shin et al. [11] considered the heat transfer behavior of a temperature dependent non-Newtonian fluid with Reiner–Rivlin model in a 2:1 channel. Hady and Gorla [12], investigated the effect of uniform suction or injection on flow and heat transfer from a continuous surface in a parallel free stream of viscoelastic second-order fluid. Jordan and Puri [13] considered Stokes' first problem for a Rivlin–Ericksen fluid of second grade in a porous half-space under isothermal conditions. Hayat et al. [14] studied the unsteady flow of a third-grade fluid occupying the space over a wall with suction or blowing at the wall surface. The constitutive equation used by Hayat is more complex than that introduced by White and Metzner [6] since it includes all third-order terms. The fact that Lie groups are used in the analysis of the resulting governing equations is also a remarkable aspect of this study. Another interesting study is performed by Sundaravadivelu and Tso [15], who investigated the influence of viscosity variations on the forced convection flow through two types of heterogeneous porous media with isoflux boundary condition. They found that viscosity variations strongly effect the heat transfer characteristics of the porous medium.

Non-Newtonian fluid flows that are attractive to researchers are many times flows that have well known solutions in the Newtonian counterpart, i.e. flows that

* Corresponding author. Tel.: +90 212 285 31 22; fax: +90 212 285 31 39.
E-mail address: kurtcebe@itu.edu.tr (C. Kurtcebe).

doi:10.1016/j.ijheatmasstransfer.2005.05.028

Nomenclature

A_n	Rivlin–Ericksen acceleration tensors
C	specific heat
C_m	series coefficients for temperature distribution
I	unit tensor
K	viscoelastic parameter ($=\phi_1/\rho L^2$)
L	width of cooling channel (Fig. 1)
p	pressure
Pr	Prandtl number ($Pr = \phi_1 C/\kappa$)
q	nondimensional temperature
Re	Reynolds number ($Re = \rho VL/\mu$)
T	temperature
u, v	velocity components in x and y directions
V	injection velocity
x, y	Cartesian coordinates

Greek symbols

η	similarity parameter (y/L)
κ	heat conduction coefficient
σ	stress tensor
μ	Newtonian viscosity coefficient
ρ	density
τ_{ij}	components of stress tensor σ
ϕ_1	viscoelastic coefficient
ϕ_2	cross-viscosity coefficient
Φ	dissipation function
Ψ	stream function

Subscript

m	power-law index for temperature distribution

are in depth investigated in the Newtonian case. One such typical flow is the laminar flow in porous channels or half spaces and the studies mentioned above are indeed of this kind. Early studies in this area are the works of Berman [16], Yuan and Finkelstein [17], White and Barfield [18] and Terill [19], which are important from point of view of technical applications of porous channel flows. Indeed, Debruge and Han [20] presented a method for turbine cooling based on these works. Recently, Goldstein et al. [21–23] have published a review of the 1999, 2000, 2001 literature on heat transfer including sections on porous media and channel flows of Newtonian and non-Newtonian fluids. Also, Ariel [24] provided an exact solution for the flow problems of a second grade fluid through two parallel porous flows in two-dimensional and axially symmetric cases.

In the present paper, the laminar flow of a second grade viscoelastic fluid between two parallel plates, one of which is externally heated and cooled by coolant injection through the other plate, is taken into consideration. The aim of the present study is to investigate two physical aspects of the flow described above, namely: the behavior of the wall friction, which is a measure for the power required for cooling, and the cooling performance depending on the viscoelastic or viscoinelastic coefficient since these two properties make up the overall performance of the cooling process.

2. Statement of the problem and governing equations

In this section, the steady-state laminar flow of a viscoelastic fluid between two parallel flat plates is considered as depicted in Fig. 1. The wall that coincides with the x -axis is heated externally and from the other perforated wall viscoelastic fluid is injected uniformly in order to cool the heated wall and the y-axis is perpendicular to the x-axis. In this perspective the flow field may be assumed to be stagnation flow.

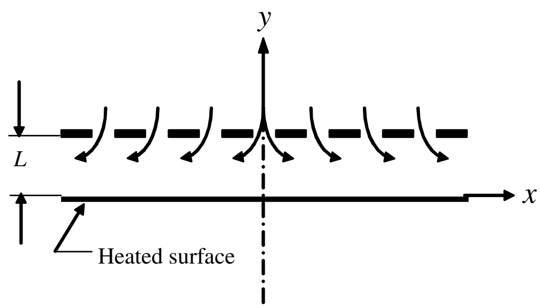

Fig. 1. Analytical model of channel flow.

If the flow is considered steady and two-dimensional the following governing equations can be written:

Continuity

$$\nabla \cdot \vec{V} = 0 \tag{1}$$

Momentum

$$\rho \frac{D\vec{V}}{Dt} = \nabla \cdot \sigma \tag{2}$$

Energy

$$\rho C \frac{DT}{Dt} = \kappa \nabla^2 T + \Phi \tag{3}$$

The most extensively investigated model for viscoelastic media is the second-order fluid model suggested by Rivlin and Ericksen [4]. For this kind of fluids the simplest constitutive equation is given by

$$\sigma = -pI + \mu A_1 + \phi_1 A_2 + \phi_2 A_1^2 \tag{4}$$

A_1 and A_2 are acceleration tensors given by

$$A_1 = \nabla\vec{V} + (\nabla\vec{V})^{\mathrm{T}} \tag{5}$$

$$A_2 = \frac{\mathrm{d}A_1}{\mathrm{d}t} + A_1(\nabla\vec{V}) + (\nabla\vec{V})^{\mathrm{T}} A_1 \tag{6}$$

where d/dt denotes material time derivative, and ∇ gradient operator and $()^{\mathrm{T}}$ transpose operator.

Recent theoretical investigations by Dunn and Fosdick [25] and Fosdick and Rajagopal [26] have indicated that for an exact model satisfying the Clauss–Duhen inequality and the assumption that the specific Helmholtz free energy be a minimum in equilibrium, the following conditions must hold:

$$\mu \geqslant 0, \quad \phi_1 \geqslant 0, \quad \phi_1 + \phi_2 = 0 \tag{7}$$

For the solution of the present heat transfer problem firstly the velocity field must be determined. Then the solution of the energy equation can be performed. For the velocity field it is convenient to define a stream function so that the continuity equation is satisfied

$$\psi = xVf(\eta) \tag{8}$$

Using Eqs. (2), (4)–(7) and eliminating the pressure term from the momentum equation the following expression can be obtained:

$$f'f'' - ff''' = \frac{f^{iv}}{Re} + K(f'f^{iv} - ff^{v}) \tag{9}$$

with the boundary conditions

$$f(0) = f'(0) = f'(1) = 1 - f(1) = 0 \tag{10}$$

Eq. (9) only contains elastic terms resulting from the A_2. The terms resulting from A_1^2 are automatically equal to zero. Eq. (9) is a nonlinear equation and difficult to solve. In order to overcome this difficulty it is convenient to apply a perturbation method. In the case of viscoelastic fluid flow, assuming $K \ll 1$ it is possible to write

$$f = f_0 + Kf_1 + K^2 f_2 + \cdots \tag{11}$$

Inserting Eq. (11) into Eq. (9) gives the following perturbation relations:

$$\frac{f_0^{iv}}{Re} - f_0' f_0'' + f_0 f_0''' = 0 \tag{12}$$

$$\frac{f_1^{iv}}{Re} - f_0' f_1'' - f_1' f_0'' + f_0 f_1''' + f_1 f_0''' = f_0 f_0^{v} - f_0' f_0^{iv} \tag{13}$$

$$\frac{f_2^{iv}}{Re} + f_0 f_2''' - f_0' f_2'' - f_2' f_0'' + f_2 f_0''' + f_1 f_1''' - f_1' f_1'' + f_0' f_1^{iv} + f_1' f_0^{iv} - f_0 f_1^{v} - f_1 f_0^{v} = 0 \tag{14}$$

with the following boundary conditions:

$$f_0(0) = f_0'(0) = f_0'(1) = f_0(1) - 1 = 0 \tag{15}$$

$$f_i(0) = f_i'(0) = f_i'(1) = f_i(1) = 0 \quad (i = 1, 2, 3, \ldots) \tag{16}$$

In order to simplify the analysis of the performance of the cooling scheme, it is assumed that the porous plate ($y = L$) has the same temperature as that of the incoming coolant, namely T_0. The heated wall is assumed to have a polynomial variation symmetrical about $x = 0$

$$T_{\mathrm{w}} = T_0 + \sum_{m=0} C_m (x/L)^m \tag{17}$$

In this perspective, at a distance y form the wall, the temperature of the fluid can be expressed as:

$$T = T_0 + \sum_{m=0} C_m (x/L)^m q_m(\eta) \tag{18}$$

Neglecting dissipative effects and the conduction flux along the x-direction and introducing Eqs. (8) and (18) into Eq. (3), one obtains the following equation containing terms of various powers of x ($m = 0, 1, 2, \ldots$)

$$mf'q_m - fq_m' = \frac{1}{Pr\,Re} q_m'' \quad (m = 0, 1, 2, 3, \ldots) \tag{19}$$

with boundary conditions

$$q_m(0) = 1, \quad q_m(1) = 0 \tag{20}$$

3. Numerical solution and results

The perturbation equations (12)–(14) obtained in the previous section, have been numerically solved using the fourth-order Runge–Kutta–Gill method. Since the equations to be solved are of fourth-order, the values of f_i, f_i', f_i'', f_i''' at the starting point of integration ($\eta = 0$ in this case) are needed. According to the given boundary conditions (15) and (16) the values of f_i'', f_i''' are unknown. Their values are determined by the shooting method. This is done as follows: two arbitrary values are assigned to $f_i''(0)$ and $f_i'''(0)$. Then the integration is performed starting from $\eta = 0$ up to $\eta = 1$. That values for $f_i''(0)$ and $f_i'''(0)$ were assumed to be accurate enough for which the following inequalities were satisfied:

$$|f_{0,\text{calculated}}(1) - 1| \leqslant 10^{-5} \quad \text{and}$$

$$|f_{i,\text{calculated}}(1)| \leqslant 10^{-5} \quad (i = 1, 2, 3) \tag{21}$$

$$|f'_{i,\text{calculated}}(1)| \leqslant 10^{-5} \quad (i = 0, 1, 2, 3, \ldots) \tag{22}$$

Following this procedure solutions have been obtained for various Reynolds numbers ranging from 0.25 to 50 for which the missing initial values are given in Table 1. The fact that the initial conditions for the second and third-order perturbation terms grow with increasing Reynolds puts a limitation on the value of the viscoelastic parameter K. Table 1 is not only important from point of view of accuracy of the solution but also due to the fact that it contains information on the variation of the wall friction parameter $f''(0)$ with the Reynolds number, namely

Table 1
Initial values for the solution of momentum equation

Re	$f_0''(0)$	$f_0'''(0)$	$f_1''(0)$	$f_1'''(0$	$f_2''(0)$	$f_2'''(0)$
0.25	6.1141	−12.58185	−0.02829	0.1362	0.02009	−0.074060
0.5	6.22789	−13.17031	−0.11889	0.57495	0.1611	−0.60918
1.0	6.4542	−14.3657	−0.5193	2.54033	1.32199	−5.2398
5.0	8.1738	−24.5833	−18.5207	104.37501	246.9776	−1302.0718
10.0	10.0367	−38.10206	−78.2745	525.60994	2241.2134	−15028.64306
30.0	15.371	−92.29445	−533.2404	5452.02574	42363.5649	−470219.37423
50.0	19.1794	−145.45262	−1197.6284	15340.61668	155687.0579	−2205475.0910

$$\tau_{xy} = \mu \frac{xV}{L^2} f''(0) \tag{23}$$

In order to find out the effect of the second-order perturbation term on the wall friction parameter some calculated values for various Reynolds numbers have been given in Table 2. According to the table the first-order term has a decreasing effect on the wall friction whereas the second-order terms does modify first-order solution.

A graphical representation of the variation of $f''(0)$ with the Reynolds number is given in Fig. 2 for various values of K. In this figure, for $f''(0)$, only the first-order perturbation term is considered. One can observe that for values of $K < 0.03$, $f''(0)$ increases with the Reynolds number and for $K > 0.03$ there is a maximum point value in the $f''(0)$–Re curve. In addition $f''(0)$ decreases with increasing value of K. It can be concluded that $f''(0)$ certainly decreases with increasing K values which is favorable from point of view of reducing friction force. This figure also reveals that the maximum allowable value for the viscoelastic parameter in order to obtain realistic results is at the order of hundreds in the interval $Re < 10$.

Table 2
Values of $f''(0)$ for various values of Re and $K = 0.02$

Re	$f_0''(0)$	$f_0''(0) + Kf_1''(0)$	$f_0''(0)+Kf_1''(0)+K^2f_2''(0)$
0.25	6.1141	6.113534	6.113542
0.5	6.22789	6.225512	6.225577
1.0	6.4542	6.443814	6.444343
5.0	8.1738	7.803386	7.902177
10.0	10.0367	8.47121	9.367695

The results for the velocity field are shown in Fig. 3. The variation of the x-component of the velocity vector is given for various values of the Reynolds number. Two interesting observations can be made. At low Reynolds numbers the velocity profiles exhibit centerline symmetry indicating a Poiseuille flow. At higher Re numbers the maximum velocity point is shifted to the solid wall where shear stress becomes larger as the Re number grows. This behavior of the velocity variation remains unchanged for $K > 0$. The dependency of the velocity variation on the viscoelastic parameter is such that the effect of the viscoelastic coefficient becomes stronger for greater Reynolds numbers. For $Re = 1$ the viscoelastic parameter K has not much influence, i.e. Newtonian and viscoelastic case ($K = 0.02$) practically coincide.

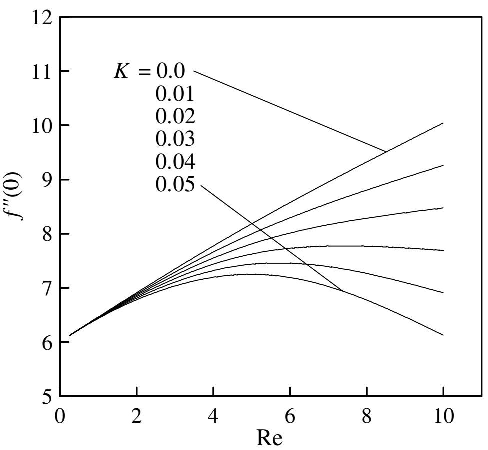

Fig. 2. $f''(0)$ for various values of K.

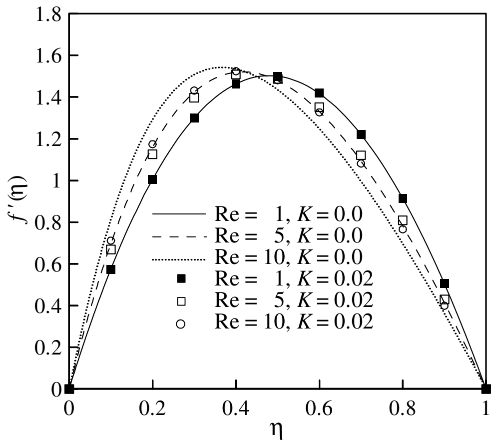

Fig. 3. $f'(\eta)$ vs. for various Re and K.

The solution of the energy equation is also obtained by applying the Runge–Kutta method described above. The criterion for convergence is taken as

$$|q_{m,\text{calculated}} - q_m(0)| \leqslant 10^{-5}$$

The cooling performance is presented in terms of the nondimensional Nusselt number. However, it should be pointed out here that it is not possible to obtain a single value for the heat transfer coefficient along the heated wall if the wall temperature follows a polynomial variation, unless the temperature profile along the blade surface is expressed by a single term in Eq. (17), i.e.

$$T_w = T_0 + C_m(x/L)^m \tag{24}$$

In this case a heat transfer coefficient h_m can be defined as

$$h_m(T_w - T_0) = -\kappa(\partial T/\partial y) \tag{25}$$

and the nondimensional Nusselt number is than obtained as:

$$\text{Nu}_m = (h_m L/\kappa) = -q'_m(0) \tag{26}$$

Taking the Prandtl number, viscoelastic parameter K and the power law index m as parameters, the variation of the Nusselt number with the Reynolds number is illustrated in Fig. 4. The specific values of the parameters number were chosen as an example. It can be observed that the power law index m, which represents the temperature variation on the heated wall and is not a property of the cooling fluid, has an increasing effect on the Nusselt number. However, we are rather interested in the influence of properties of the coolant fluid on the heat transfer, namely, its viscoelasticity and Prandtl number. The Prandtl number obviously has a strongly increasing effect. The same is not true for the viscoelastic parameter K, which clearly reduces the Nusselt number when compared with Newtonian fluid flow.

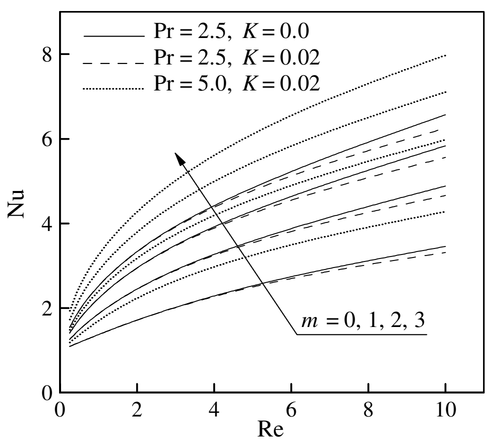

Fig. 4. *Nu* vs. *Re* effect of *K* and *Pr*.

4. Conclusion

In this paper, the flow of a non-Newtonian viscoelastic fluid between two parallel flat plates is studied. The flow occurs between two parallel plates one of which is externally heated. In order to cool the heated plate coolant fluid is injected from the other plate, which is perforated. Two aspects of the flow are investigated, namely, (i) wall friction, which is a measure for the energy required for coolant injection (ii) heat transfer characteristics. It is found out that the wall friction decreases in case a viscoelastic coolant is used. In contrast, the cooling performance, i.e. the Nusselt number, becomes worse with respect to Newtonian fluid.

References

[1] G. Böhme, Non-Newtonian Fluid Mechanics, North Holland, Amsterdam, 1981.
[2] R.R. Huilgol, N. Phan-Thien, Fluid Mechanics of Viscoelasticity, Elsevier, Amsterdam, 1997.
[3] M. Akcay, A. Yukselen, Drag reduction of a non-Newtonian fluid by fluid injection on a moving wall, Arch. Appl. Mech. 69 (1999) 215–225.
[4] R.S. Rivlin, J.L. Ericksen, Stress–deformation relations for isotropic materials, J. Ration. Mech. Anal. 4 (1955) 323–425.
[5] R.S. Rivlin, Further remarks on the stress–deformation relations for isotropic materials, J. Ration. Mech. Anal. 4 (1955) 681–702.
[6] J.L. White, A.B. Metzner, Constitutive equations for viscoelastic fluids with application to rapid external flows, AIChE J. 11 (2) (1965) 324–330.
[7] J.E. Dunn, K.R. Rajagopal, Fluids of differential type: critical review and thermodynamic analysis, Int. J. Eng. Sci. 33 (1995) 689–747.
[8] A.C. Srivastava, The flow of a non-Newtonian liquid near a stagnation point, ZAMP 9 (1958) 80–84.
[9] G.K. Rajeswari, S.L. Rathna, Flow of a particular class of non-Newtonian visco-elastic and visco-inelastic fluids near a stagnation point, ZAMP 13 (1962) 43–57.
[10] S. Raghay, A. Hakim, Numerical simulation of White–Metzner fluid in a 4:1 contraction, Int. J. Numer. Fluids 35 (2001) 559–573.
[11] S. Shin, H.H. Ahn, Y.I. Cho, C.H. Sohn, Heat transfer behavior of a temperature dependent non-Newtonian fluid with Reiner–Rivlin model in a 2:1 rectangular duct, Int. J. Heat Mass Transfer 42 (1999) 2935–2942.
[12] F.M. Hady, R.S.R. Gorla, Heat transfer from a continuous surface in a parallel free stream of viscoelastic fluid, Acta Mech. 128 (1998) 201–208.
[13] P.M. Jordan, P. Puri, Stokes' first problem for a Rivlin–Erickssen fluid of second grade in a half porous space, Int. J. Non-linear Mech. 38 (2003) 1019–1025.
[14] T. Hayat, A.H. Kara, E. Mononiat, Exact flow of a third-grade fluid on a porous wall, Int. J. Non-linear Mech. 38 (2003) 1533–1537.
[15] K. Sundaravadivelu, C.P. Tso, Influence of viscosity variations on the forced convection flow through two

types of heterogeneous porous media with isoflux boundary condition, Int. J. Fluid Mech. 46 (2003) 2329–2339.
[16] A.S. Berman, Laminar flow in channels with porous walls, J. Appl. Phys. 24 (1953) 1232–1235.
[17] S.W. Yuan, A.B. Finkelstein, Laminar pipe flow with injection and suction through a porous wall, Trans. ASME 78 (May) (1956) 719–724.
[18] F.M. White, B.F. Barfield, Laminar flow in a uniformly porous channel, J. Appl. Mech., ASME 80 (December) (1958) 613–617.
[19] R.M. Terill, Laminar flow in a uniformly porous channel with large injection, Aeronaut. Q. 16 (1965) 322–332.
[20] L.L. Debruge, L.S. Han, Heat transfer in a channel with a porous wall for turbine cooling application, J. Heat Transfer, Trans. ASME (1972) 385–390.
[21] R.J. Goldstein et al., Heat transfer—a review of 1999 literature, Int. J. Heat Mass Transfer 44 (2001) 3579–3699.
[22] R.J. Goldstein et al., Heat transfer—a review of 2000 literature, Int. J. Heat Mass Transfer 45 (2002) 2853–2957.
[23] R.J. Goldstein et al., Heat transfer—a review of 2001 literature, Int. J. Heat Mass Transfer 46 (2003) 1887–1992.
[24] P.D. Ariel, On exact solutions of flow problems of a second grade fluid through two parallel porous walls, Int. J. Eng. Sci. 40 (2002) 913–941.
[25] J.E. Dunn, R.L. Fosdick, Thermodynamics stability and boundedness of fluids of complexity 2 and fluids of second grade, Arch. Rat. Mech. Anal. 56 (1974) 191–252.
[26] R.L. Fosdick, K.R. Rajagopal, Anomalous features in the model of second-order fluids, Arch. Rat. Mech. Anal. 70 (1979) 145–152.